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Final Exam, Fr 25.6. 2021

1. Feynman Kac (3 points) Consider the solution X of the SDE

$$dX_t = (\theta - X_t)dt + \sigma\sqrt{X_t(1-X_t)}dW_t$$

for parameters $\sigma > 0$ and $\theta \in (0,1)$. (It is known that a unique solution exists for every initial value $0 \le x \le 1$ and that the solution is an element of $[0,1]$ for all t.) Define for parameters $\rho_0, \rho_1 > 0$ the function

$$F(t,x) = E_x\Big(\exp\big(-\int_0^{T-t} \rho_0 + \rho_1 X_s ds\big)\Big), \quad (t,x) \in [0,T] \times [0,1]\,.$$

Use the Feynman Kac formula to derive a terminal value problem for F.

2. Uniqueness of semimartingale decomposition. (3 points) Consider a continuous semimartingale

$$X_t = X_0 + M_t + A_t\,, \quad t \ge 0,$$

where M is a local martingale with continuous paths and where $A_t = \int_0^t a_s ds$ is a process of finite variation. Show that this decomposition is unique, that is for any other decomposition $X_t = X_0 + \widetilde{M}_t + \widetilde{A}_t$ with these properties it holds that $M = \widetilde{M}$ and $A = \widetilde{A}$.

3. Black Scholes model and binary option. (5 points) Consider in the context of the Black Scholes model with stock price dynamics $dS_t = \mu S_t dt + \sigma S_t dW_t$, initial stock price $S_0 > 0$ and with money market account $B_t = \exp(rt)$ for $r > 0$ a so-called binary put option with payoff $h(S_T) = 1_{\{S_T \le K\}}$ for some $K > 0$. Use the risk neutral pricing formula to compute the price of the option at $t = 0$ and the stock position of the corresponding hedging portfolio.

4. Volatility skew and Black Scholes model. (4 points) Briefly explain what is meant by the *skew pattern of implied volatility* or *implied volatility skew* on options' markets. Discuss how the implied volatility skew is related to empirical deficiencies of the Black Scholes model.

Good luck!